Finite Math - Spring 2017

Lecture Notes - 3/17/2017

Homework

- Section 4.3 - 1, 4, 6, 7, 9, 10, 11, 14, 16, 41, 42, 44, 45, 48, 52, 54, 57, 58, 59, 73, 76

Section 4.3 - Gauss-Jordan Elimination

Example 1. *Solve by Gauss-Jordan elimination:*

$$\begin{array}{rcrcrcl} 3x_1 & + & 5x_2 & - & x_3 & = & -7 \\ x_1 & + & x_2 & + & x_3 & = & -1 \\ 2x_1 & & & + & 11x_3 & = & 7 \end{array}$$

Solution. $x_1 = -2, x_2 = 0, x_3 = 1$

Example 2. *Solve by Gauss-Jordan elimination:*

$$\begin{array}{rcrcrcl} 3x_1 & - & 4x_2 & - & x_3 & = & 1 \\ 2x_1 & - & 3x_2 & + & x_3 & = & 1 \\ x_1 & - & 2x_2 & + & 3x_3 & = & 2 \end{array}$$

Solution. *No solution.*

Example 3. *Solve by Gauss-Jordan elimination:*

$$\begin{array}{rcrcrcl} 3x_1 & - & 4x_2 & - & x_3 & = & 0 \\ 2x_1 & - & 3x_2 & + & x_3 & = & 1 \\ x_1 & - & 2x_2 & + & 3x_3 & = & 2 \end{array}$$

Solution. $x_1 = 7t - 4, x_2 = 5t - 3, x_3 = t$

Example 4. *Solve by Gauss-Jordan elimination:*

$$\begin{array}{rcrcrcl} 2x & - & y & - & 3z & = & 8 \\ x & - & 2y & & & = & 7 \end{array}$$

Solution. *The augmented matrix is*

$$\left[\begin{array}{ccc:c} 2 & -1 & -3 & 8 \\ 1 & -2 & 0 & 7 \end{array}\right]$$

Begin as always, by getting the 1 *in the top left*

$$\left[\begin{array}{ccc:c} 2 & -1 & -3 & 8 \\ 1 & -2 & 0 & 7 \end{array}\right] \overset{R_1 \leftrightarrow R_2}{\sim} \left[\begin{array}{ccc:c} 1 & -2 & 0 & 7 \\ 2 & -1 & -3 & 8 \end{array}\right]$$

Then getting the zero below it

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 7 \\ 2 & -1 & -3 & 8 \end{array}\right] \overset{R_2-2R_1\to R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 7 \\ 0 & 3 & -3 & -6 \end{array}\right]$$

Now we get the 1 *in the second column*

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 7 \\ 0 & 3 & -3 & -6 \end{array}\right] \overset{\frac{1}{3}R_2\to R_2}{\sim} \left[\begin{array}{ccc|c} 1 & -2 & 0 & 7 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

then use this to get a zero above it

$$\left[\begin{array}{ccc|c} 1 & -2 & 0 & 7 \\ 0 & 1 & -1 & -2 \end{array}\right] \overset{R1+2R_2\to R_1}{\sim} \left[\begin{array}{ccc|c} 1 & 0 & -2 & 3 \\ 0 & 1 & -1 & -2 \end{array}\right]$$

This tells us that $x - 2z = 3$ *and* $y - z = -2$*. Since* z *is in both equations, we will let* $z = t$*, then we have* $x = 2t + 3$ *and* $y = t - 2$*. So the solutions is*

$$x = 2t + 3, y = t - 2, z = t$$

for real numbers t*.*

Example 5. *Solve by Gauss-Jordan elimination:*

$$\begin{array}{rcrcrcr} 2x_1 & + & 4x_2 & - & 6x_3 & = & 10 \\ 3x_1 & + & 3x_2 & - & 3x_3 & = & 6 \end{array}$$

Solution. $x_1 = -t - 1, x_2 = 2t + 3, x_3 = t$

Example 6. *A company that rents small moving trucks wants to purchase* 16 *trucks with a combined capacity of* 19,200 *cubic feet. Three different types of trucks are available: a cargo van with a capacity of* 300 *cubic feet, a* 15*-foot truck with a capacity of* 900 *cubic feet, and a* 24*-foot truck with a capacity of* 1,500*-cubic feet. How man of each type should the company purchase?*

Solution. $t - 8$ *cargo vans,* $-2t + 24$ *of the* 15*-foot trucks, and* t *of the* 24 *foot trucks, where* $t = 8$*,* 9*,* 10*,* 11*, or* 12